

GCE

Further Mathematics A

Y535/01: Additional pure mathematics

AS Level

Mark Scheme for June 2024

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Question	Answer	Marks	AO	Guidance
1	$28A3B_{12} = 11 + (3 \times 12) + (10 \times 12^2) + (8 \times 12^3) + (2 \times 12^4)$ $= 56783$	M1 A1 [2]	1.1 1.1	DR Use of powers of 12 with $A = 10$ and $B = 11$. Condone one error in the expansion of powers of 12. Values of A and B can be implied by correct answer below.

Question	Answer	Marks	AO	Guidance
2	(a) Attempt at vector product of their a and b: $\begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$ $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 60 \\ 15 \\ -20 \end{pmatrix}$ $ \mathbf{a} \times \mathbf{b} = \sqrt{60^2 + 15^2 + (-20)^2}$ $\text{Area } \triangle OAB = \frac{1}{2} \mathbf{a} \times \mathbf{b} = \frac{65}{2} \text{ (square units)}$	M1* A1 M1dep* A1	1.1 1.1 1.1 1.1	Must be clear that it is the vector product being attempted. Can be implied by two correct components. If using Area $\triangle OAB = \frac{1}{2} \mathbf{a} \mathbf{b} \sin \theta$, attempt at $ \mathbf{a} $, $ \mathbf{b} $ and θ via the scalar product. Or $\mathbf{b} \times \mathbf{a}$ If using Area $\triangle OAB = \frac{1}{2} \mathbf{a} \mathbf{b} \sin \theta$, $ \mathbf{a} = \sqrt{101}$, $ \mathbf{b} = \sqrt{61}$, $\theta = 55.9 \dots^0$ Attempt at the magnitude of their $\mathbf{a} \times \mathbf{b}$ soi Or Area $\triangle OAB = \frac{1}{2} \sqrt{101} \times \sqrt{61} \sin 55.9 \dots^0$ Triangle formula used CAO
		[4]		

Question		Answer	Marks	AO	Guidance
2	(b)	$\begin{pmatrix} 2 \\ 4 \\ 3\lambda \end{pmatrix} \times \begin{pmatrix} \lambda \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 24+12\lambda \\ 3\lambda^2-12 \\ -8-4\lambda \end{pmatrix} = \mathbf{0}$ $\lambda = -2$	M1 <		

Question		Answer	Marks	AO	Guidance
3	(a)	$f_x = 8xy - 6y^2 - \frac{1}{3}x^3$	B1	1.1	
		$f_y = 4x^2 - 12xy$	B1	1.1	
		$f_y = 0 \Rightarrow x = 3y$ (Noting $x \neq 0$ not required)	M1	1.1	Establishing a relationship between x and y by setting one first partial derivative to zero
		$f_x = 0 \Rightarrow 8.3y.y - 6y^2 - \frac{1}{3}.27y^3 = 0$	M1	1.1	Substituting correctly for first variable in second partial derivative set to zero and solving attempt of some cubic polynomial
		$\Rightarrow b = 2$	A1	1.1	Second variable value CAO
		$\Rightarrow a = 6$ and $c = 36$	A1	1.1	Both remaining values correct Condone x, y, z for a, b, c throughout

Question			Answer	Marks	AO	Guidance
				[6]		
	(b)	i	At U_1 , $x = 5.9, y = 2$, $f_x = 1.94 (= 8 \times 5.9 \times 2 - 6 \times 2^2 - \frac{1}{3} \times 5.9^3)$ At U_2 , $x = 6.1, y = 2$, $f_x = -2.06 (= 8 \times 6.1 \times 2 - 6 \times 2^2 - \frac{1}{3} \times 6.1^3)$	B1 B1 [2]	1.1 1.1	If B0B0 , SCB1 for both correct substitutions
		ii	At V_1 , $x = 6, y = 1.9$, $f_y = 7.2 (= 4 \times 6^2 - 12 \times 6 \times 1.9)$ At V_2 , $x = 6, y = 2.1$, $f_y = -7.2 (= 4 \times 6^2 - 12 \times 6 \times 2.1)$	B1 B1 [2]	1.1 1.1	If B0B0 , SCB1 for both correct substitutions
		iii	$\cap$ -shaped parabola on an x - z set of axes labelled or implied by an equation in (x,z) or $(x, f(x,2))$ $\cap$ -shaped parabola on a y - z set of axes labelled or implied by an equation in (y,z) or $(y, f(6,y))$	B1 B1 [2]	1.1 1.1	Condone positional vaguenesses Ditto

Question	Answer	Marks	AO	Guidance
4	(a) $F_{n+1} = F_n + F_{n-1}$ $F_{n+4} = F_{n+3} + F_{n+2}$ $F_{n+2} = 2F_n + F_{n-1}$ $= 2F_{n+2} + F_{n+1}$ $F_{n+3} = 3F_n + 2F_{n-1}$ $= 3F_{n+1} + 2F_n$ or $F_{n+4} = 5F_n + 3F_{n-1}$	M1	2.1	Fibonacci recurrence relation used correctly at least twice
		A1	2.2a	CAO If M0, SCB1 for trial and error with F_r and F_s checked for $r, s \geq 6$
	(b) When $n = 4$, $F_4 = 3$ which is a multiple of 3 so result is true (or $n = 8$, $F_8 = 21$) Assume the result is true for $n = k$, where k is a multiple of 4 (or $n=4k$) Therefore $F_k = 3N$ (or $F_{4k} = 3N$) where N is a positive integer or F_k (or F_{4k}) is a multiple of 3 Using part (a), $F_{k+4}(F_{4k+4}) = (their\ 5)\ F_r + (their\ 3)\ F_s$, r, s in terms of k, $4k - 4 < r, s < 4k + 4$, $r \neq s$ $F_{k+4} = 3(5N + F_{k-1})$ or $F_{4k+4} = 3(5N + F_{4k-1})$ which is also a multiple of 3 Since $F_4 = 3$ is a multiple of 3 and F_k a multiple of 3 when k is a multiple of 4 $\Rightarrow F_{k+4}$ a multiple of 3, it follows by induction that F_n is always a multiple of 3 if n is a multiple of 4	B1	1.1	Basic case
		M1	2.1	Clearly reasoned inductive step (no need to note that all terms are integers)
		A1	2.4	Fully correct conclusion with consistent use of subscripts www

Question	Answer	Marks	AO	Guidance
		[3]		

Question	Answer	Marks	AO	Guidance
5	(a) $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1 [1]	1.2	No need to observe that $\det(\mathbf{I}) = 1$ Accept I
	(b) G consists of the powers of $\mathbf{P}$, which has order 6 ($n = 6$)	B1 B1 [2]	2.4 2.2a	oe such as $\mathbf{P}^6 = \mathbf{I}$. Can be implied by explicit calculation of $\mathbf{P}^6$. Or recognising that $\mathbf{P}$ is a clockwise rotation through 60° . Allow unsupported statement and without mention that no lower power of $\mathbf{P}$ gives I .
	(c) i $\mathbf{Q}^2 = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix} (= \mathbf{RQR})$ $\mathbf{QR} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ or $\mathbf{RQ} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ $\mathbf{QR} = \mathbf{RQ}^2$ and $\mathbf{RQ} = \mathbf{Q}^2\mathbf{R}$ $\mathbf{Q} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, I and $m = 6$	B1 B1 B1 B1	1.1 1.1 1.1 2.2a	All six elements must be seen (explicitly as matrices or in terms of Q and R) and no others for this B1 . (Condone failure to note that $\mathbf{Q}^3 = \mathbf{R}^2 = \mathbf{I}$.)

Question		Answer	Marks	AO	Guidance
			[4]		
	(c) ii	One of $\{\mathbf{R}, \mathbf{I}\}$ (or $\{\mathbf{R}, \mathbf{R}^2\}$), $\{\mathbf{Q}, \mathbf{Q}^2, \mathbf{I}\}$ (or $\{\mathbf{Q}, \mathbf{Q}^2, \mathbf{Q}^3\}$), $\{\mathbf{QR}, \mathbf{I}\}$, $\{\mathbf{RQ}, \mathbf{I}\}$ Another correct subgroup. All correct and no others.	B1 B1 B1 [3]	2.2a 1.2 1.1	One correct subgroup Ignore $\{\mathbf{I}\}$ and/or H if mentioned. Alternative forms may be used or matrices given instead. Condone absence of brackets.
5	(d)	True: Cyclic $\Rightarrow$ Abelian, so G is Abelian or All powers of a single element commute $\Rightarrow$ Abelian True: G is cyclic, generated by $\mathbf{P}$ False: H is not Abelian, since (e.g.) $\mathbf{QR} \neq \mathbf{RQ}$ False: Not Abelian $\Rightarrow$ not cyclic, so H is not cyclic or There is no generator in H	B1 B1 B1 B1 [4]	2.4 2.5 2.2a 2.4	Each answer must have a valid reason supporting it

Question			Answer	Marks	AO	Guidance
6	(a)	i	$n = 9k + 3$ and $n = 9k + 6$ or $n \equiv 3 \pmod{9}$ and $n \equiv 6 \pmod{9}$	B1 [1]	1.2	Allow $n = 9k \pm 3$ Condone $9n + 3$ and $9n + 6$ (or $9n - 3$), not $n = 9k + r$
		ii	For $n = 9k + 3$, $f(n) = 1 + 2^{9k+3} + 4^{9k+3} = 1 + 8.(2^9)^k + 64.(2^9)^{2k}$ $2^9 = 512 \equiv 1 \pmod{73}$ so that $f(n) \equiv 1 + 8.(1)^k + 64.(1)^{2k} \pmod{73}$ $= 1 + 8 + 64 = 73 \equiv 0 \pmod{73}$ and $f(n)$ is a multiple of 73 Similarly, for $n = 9k + 6$, $f(n) = 1 + 64.(2^9)^k + 4096.(2^9)^{2k}$ so that $f(n) \equiv 1 + 64.(1)^k + 8.(1)^{2k} = 1 + 64 + 8 = 73 \equiv 0 \pmod{73}$ and $f(n)$ is a multiple of 73	M1 B1 M1 A1 M1 A1 [6]	3.1a 1.1 3.1a 3.2a 1.1 3.2a	Writing 2^{r+3} as $2^r \times 2^3$ and 4^{r+3} as $4^r \times 4^3$ Noted or used at any stage NB $262144 = 512^2$ Considering $f(n) \pmod{73}$ by substituting $(2^9)^k$ and $(2^9)^{2k}$ with 1 All correctly concluded Writing 2^{r+6} as $2^r \times 2^6$ and 4^{r+6} as $4^r \times 4^6$ (or suitably modified for $n = 9k - 3$) From fully correct working, including noting that $4096 \equiv 8 \pmod{73}$ NB $n = 9k + 6$ could be considered first.
	(b)		If $n = 9k$, then $f(n) = 1 + 512^k + (512^2)^k$ or $1 + 512^k + (262144)^k$ $\equiv 1 + 1^k + (1^2)^k = 3 \pmod{73}$	M1 A1 [2]	1.1 2.2a	Correct set up using indices CAO It must be clear that this is always the case If M0 , SC1 substituting $n=9$ and evaluating mod 73 to find 3: $f(9) = 1 + 2^9 + 4^9 (=262657) \equiv 3 \pmod{73}$

Question	Answer	Marks	AO	Guidance
7	(a) If $E_0 = H_0 = 1200$ and $H_{n+1} = \left(1 - \frac{1}{8}\right) H_n$, then $H_{n+6} = \left(\frac{7}{8}\right)^6 H_n$ and $E_{n+1} = \left(\frac{7}{8}\right)^6 E_n$ plus a ‘boost’ of 500	B1 B1 [1]	3.3 2.4	H_n being the hourly amount of enzyme Adequately explained in words and/or symbols.
	(b) Gen. Soln. is given by $(E_n =)$ CS (+) PS $CS = a\left[\left(\frac{7}{8}\right)^6\right]^n$ $PS = \frac{500}{1 - \left(\frac{7}{8}\right)^6}$ Use of $E_0 = 1200$ to evaluate their “a” $E_n = \left(1200 - \frac{500}{1 - \left(\frac{7}{8}\right)^6}\right) \times \left(\frac{7}{8}\right)^{6n} + \frac{500}{1 - \left(\frac{7}{8}\right)^6}$	M1 A1 A1 M1 A1 [5]	1.1 1.1 1.1 3.4 1.1	Including good attempts at $CS = a \times r^n$ (with 1 arbitrary constant) and $PS = \text{constant}$ Must be $E_n = CS + PS$ PS must be numerical Allow approximate numerical equivalents; e.g. $292.9 \times 0.4488^n + 907.1$
	(c) $\left(\frac{7}{8}\right)^n$ or $\left(\frac{7}{8}\right)^{6n} \rightarrow 0$ as $n \rightarrow \infty$ $\Rightarrow E_n \rightarrow 907$	B1 B1 [2]	2.4 2.2b	Condone explanation that $\left(\frac{7}{8}\right)^n$ or $\left(\frac{7}{8}\right)^{6n}$ becomes small (negligible) as $n \rightarrow \infty$. Correct to 3sf

Question	Answer	Marks	AO	Guidance
7	(d) From $H_0 = 1200$, the hourly sequence H_n runs 1050, 918.75, 803.91, 703.42, ... Attaining $H_6 = 538.55 \rightarrow E_1 = \mathbf{1038(.55)}$ Continuing one hour at a time; so $H_7 = 908.7$, $H_8 = 795.14$, $H_9 = 695.75$, $H_{10} = 608.78$, $H_{11} = 532.68$, $H_{12} = \mathbf{466.10}$ Explaining that, just before the second 500 (mg) is added, the amount of enzyme has fallen below the required minimum amount, so the experiment's validity has failed	M1 A1 A1 [3]	3.1a 3.5a 3.5b	DR Breaking the problem down into single 1-hour steps, multiplying by 0.875 each time. Making the jump from the end of the first six-hour period to the start of the second. Sight of 1038(.55) or $1200 \times \left(\frac{7}{8}\right)^6 + 500$ FT their solution from part (b) Continue as before to H_{12}. Sight of 466(.10) with (numerical) justification. FT their solution from part (b) if their $H_{12} < 500$ ‘466<500 so requirement not met’

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